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Abstract

Motivated by the potential tension between coordination, which may require discriminating among identical workers, and social comparisons, which may intuitively call for small pay differentials, we analyze the design of optimal rewards in an organization with inequality-averse workers whose tasks are complementary. Inequality aversion surprisingly results in higher monetary incentives and may also yield more inequality among agents. We also show that disadvantageous inequality aversion is of first-order importance compared to advantageous inequality aversion, a result that is consistent with existing evidence. Moreover, the distribution of rewards may be non monotonic, with the most inequality-averse agents lying at both ends of the distribution. Our analysis also sheds light on the crucial role of coordination, as most results would be reversed if the principal could costlessly select her most preferred equilibrium outcome.

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1 Introduction

While social comparisons seem to strongly affect workers well-being and performance, little is known on how organizations should account for these features. Should inequality aversion yield a decrease in reward inequalities within organizations? Should it be associated with lower monetary incentives? More generally, how should social comparisons affect the distribution of rewards within organizations?

In order to address these questions, we analyze the implications for optimal contract design of inequality aversion. Specifically, when workers' decisions in an organization exhibits complementarity effects, there is a potentially important tension between social comparisons and incentives. Indeed, a given worker may fear the risk that the other workers do not perform their task, making her own effort useless. One solution is to offer a sufficiently high reward to a worker so that exerting effort is beneficial to her, even when the other workers shirk. This in turn removes the risk (for others) that this worker does not perform her task. Using this argument in a contracting setting with externalities[1], different contributions highlight that the need for coordinating agents implies that the optimal reward scheme is discriminatory, meaning that workers obtain different rewards even if they are identical (Segal, 2003; Winter, 2004). Yet, when workers are averse to inequalities, unequal rewards are likely to negatively affect them and, as such, to weaken the power of incentives.

We explore the implications of such tension and introduce an organization setting with multiple inequality-averse workers whose tasks are complementary. We use the classical model of inequality aversion introduced by Fehr and Schmidt (1999). So, there are interactions between multiple heterogeneous agents who positively value their own material payoff and negatively value the differences between their material payoff and that of the other agents. We consider the problem of a principal who seeks to ensure effort provision and coordination of the agents at least cost.

We obtain several important results. Among others, we show that inequality aversion may imply more inequality within the organization, and that it should be associated with larger monetary incentives. We further highlight that disadvantageous inequality aversion is of first-order importance compared to advantageous inequality aversion. The distribution of rewards may also be non monotonic, and the most inequality-averse agents may lie at both ends of the reward distribution.

When designing the optimal reward scheme, the principal takes into account that offering a reward to an agent has both direct and indirect effects on the other agents' decisions. The direct effect is due to social comparisons, while the indirect effect is due to the complementarity effect in the agents' efforts. While the indirect effect is clearly positive, the sign of the direct effect is not straightforward, as the level of the agent's payoff may increase or decrease when her effort increases. We first characterize the optimal reward scheme, and show that inequality aversion results in higher individual rewards compared to the case where agents do not exhibit social preferences. Indeed, when an agent exerts effort, both disadvantageous and advantageous inequalities increase

[1]The seminal paper is Segal (1999).

compared to the case where this agent shirks. Such increases must be compensated by an increase in this agent's reward when she is inequality-averse. This is an important result, which contradicts a conjecture made by Cohn et al. (2014) that inequality aversion should cause a reduction in pay inequality and that it should be associated with smaller monetary incentives. We also find that disadvantageous inequality aversion is of first-order importance compared to advantageous inequality aversion. Indeed, the optimal contract exhibits the divide and conquer property (DAC): it induces a ranking among agents such that each agent prefers to exert effort when all higher ranked agents exert effort while all lower ranked agents do not exert effort. This also results in a higher ranked agent obtaining a larger reward. When the higher ranked agents exert effort, an agent is always disadvantaged, whether she exerts effort or not. By contrast, this same agent is advantaged compared to the lower ranked agents only if she does exert effort herself: if she does not exert effort, then there is no advantageous inequality. In a sense, the requirement that cooperation be induced as a unique equilibrium results in a contract exhibiting a DAC property, which puts a lot of importance on the higher ranked agents, and as such on the disadvantageous inequality-aversion parameters.

Next, we study the optimal distribution of rewards and show that the distribution may be a non monotonic function of the degree of inequality aversion: the most averse agents may indeed lie at both ends of the reward distribution. For instance, if agents are heterogeneous in their degree of aversion to disadvantageous inequality only, their ranking is characterized by the magnitude of the disutility resulting from disadvantageous inequalities. An agent's disadvantageous inequality-aversion parameter has both a direct effect on this disutility and an indirect effect resulting from its impact on this agent's optimal bonus. The direct effect is positive, while the indirect effect is negative, and the net impact depends on the fundamentals. We then study how the existence of inequality aversion affects the inequality level within the organization. We show that inequality is unambiguously lower compared to a situation where the agents are not inequality averse. However, when the agents are inequality averse, an increase in inequality aversion may actually lead to an increase in inequality at the bottom of the reward distribution.[2]

We then conclude the analysis by examining the specific effect of the existence of a coordination problem on the optimal contract under social comparisons. Indeed, so far the principal is assumed to explicitly account for the existence of this problem when designing the contract structure: the optimal contract must induce her desired outcome as a *unique* equilibrium of the induced game. We then come back to the same problem, and instead assume that the principal can costlessly select her most preferred equilibrium outcome. As such, the principal now offers the least-cost contract that induces her desired outcome as *one* (of possibly many) equilibrium outcome. This has a notable effect on the analysis, as several qualitative conclusions are reversed in such case. Specifically, we then find that inequality aversion leads to a decrease in the individual rewards and that advantageous inequality aversion is of first order importance compared to disadvantageous

[2]Montero (2007) also shows (in a very different setting) that inequality aversion may increase inequality.

inequality aversion. The fact that cooperation be only one of several equilibria allows the principal to reduce the overall cost of the contractual scheme by relaxing the incentive constraints for all agents: this actually implies that advantageous inequality-aversion parameters then become the most relevant parameters. Moreover, the distribution of the rewards is very different from the one we obtain when the principal has to solve the coordination problem.

Literature. This paper is connected to two different strands in the literature. The first one relates to an empirical literature on the potential effect of social comparisons on workers' well-being and performance in organizations.[3] Recent empirical studies highlight that social comparisons do affect workers' pay and job satisfaction (Card et al., 2012), workers' performance (Cohn et al., 2014), output and attendance (Breza et al., 2017), and even decisions to quit (Dube et al., 2019). While social comparisons seem to strongly affect workers well-being and performance, little is known about how organizations should account for these behaviors. This is the main goal of this paper which, to our knowledge, is the first to analyze the interplay between agents' coordination problem and inequality aversion. We provide theoretical results, and as such general conclusions, about the specific characteristics of the optimal contract when multiple agents interact within an organization under social comparisons. Some of our conclusions are consistent with empirical regularities: for instance, our conclusion about the first-order importance of disadvantageous inequality aversion is consistent with experimental findings in Cohn et al. (2014).

The present contribution also relates to the literature on behavioral contract theory.[4] Specifically, this study relates to the literature focusing on optimal contracting with multiple inequality-averse agents Englmaier and Wambach (2010); Rey-Biel (2008); Demougin et al. (2006).[5] Very few papers consider a principal - multiple agent relationship under inequality aversion. Goel and Thakor (2006) analyze the case where agents envy each other: The focus is on contracts inducing surplus sharing in the case of homogeneous agents.[6] A part of this literature provides results related to team-based incentives, as for instance Bartling and Von Siemens (2010, 2011), Rey-Biel (2008), or Itoh (2004). The general focus of these studies is on how the principal tailors agents' incentives to account for agents' preferences by offering more equitable contracts or team-based incentive schemes. All these papers differ notably from the present contribution in terms of the setting considered and the research questions. As all these contributions focus on two-agent settings, none of them is suited to analyze the (possibly non monotonic) distribution of rewards. Moreover, these contributions do not tackle the situation in which a principal uses rewards to explicitly induce coordination among agents.[7]

[3] Recent laboratory experimental evidence clearly rejects the assumption that individuals care only about their material payoffs (Camerer, 2003).

[4] See Koszegi (2014), DellaVigna (2009), and Rabin (1998) for extended reviews.

[5] Englmaier and Wambach (2010) mainly consider the effect of inequality aversion on contract design in a single principal - single agent setting when the agent cares for the principal's material payoffs.

[6] Gürtler and Gürtler (2012) analyze the effect of inequality aversion on individuals' behavior in a quite general setting. Yet, the focus is on the externalities resulting from such preferences in an homogeneous population setting.

[7] Dhillon and Herzog-Stein (2009) analyze the coordination problem in a very different setting where agents are

Structure of the paper. The remainder of the contribution is organized as follows. Section 2 introduces the model. In Section 3 we characterize the optimal reward scheme that ensures effort and coordination. In Section 4 we study the impact of inequality aversion on inequality. In Section 5 we analyze the situation where the principal can costlessly solve the coordination problem. Section 6 concludes.

2 The model

We consider an organization in which a principal offers individual rewards (bilateral contracts) to several agents who can either exert effort or shirk. The environment is such that an agent's effort results in cost-reducing externalities: in other words, the agents' efforts are complementary.

An agent who decides to shirk receives his outside option c (the benefits from shirking). An agent who decides to exert effort generates a cost-reducing externality: the cost of other agents who also exert effort is reduced by magnitude $w \geq 0$. The principal aims to induce work at least cost. In order to reach this goal, she offers a reward scheme $v = (v_1, v_2, ..., v_n)$ to agents in set N, with $N = [1, 2, ..., n]$. An agent's reward is conditional on his choice: for any $i \in N$ agent i receives v_i from the principal if he exerts effort and 0 otherwise. The agents' decision vector is denoted $e = (e_1, ..., e_n) \in \{0, 1\}^n$, where $e_i = 1$ means that agent i exerts effort while $e_i = 0$ means that she decides to shirk.

The timing is as follows: first, the principal offers a reward scheme to the agents; second, the agents observe the principal's proposal and simultaneously decide whether to exert effort or not, and they receive their individual reward only if they exert effort.

Under reward scheme v agent i's material payoff can be written as follows:

$$\pi(e, v_i) = c + e_i v_i - \left(1 - \sum_{j \neq i} e_j \frac{w}{c}\right) c e_i \tag{1}$$

The first two terms on the right hand side of the equality corresponds to agent i's benefits. If she chooses to shirk she gets c while she obtains a higher benefit $c + v_i$ if she decides to exert effort. The last term on the right hand side of the equality corresponds to the cost borne by agent i. Specifically, if she chooses to shirk ($e_i = 0$) she does not bear any cost. By contrast, if she chooses to exert effort ($e_i = 1$) then her cost depends on how many other agents choose to exert effort as well. If there are $k \leq N - 1$ other agents who choose to do so, then agent i's cost of effort is given by $c \left(1 - k\frac{w}{c}\right)$: thus other effort-exerting agents generate externalities that reduce agent i's own cost from exerting effort: the agents' efforts exhibit complementarity effects.

The agents are assumed to be inequality averse *a la* Fehr and Schmidt (1999). Specifically, agent

status-seeking.

i's utility function is then:

$$U_i(e,v) = \pi_i(e,v_i) - \frac{\alpha_i}{n-1}\sum_{k\neq i}\max\{\pi_k(e,v_k) - \pi_i(e,v_i), 0\} - \frac{\beta_i}{n-1}\sum_{k\neq i}\max\{\pi_i(e,v_i) - \pi_k(e,v_k), 0\}, \tag{2}$$

where $0 \leq \beta_i < 1$ and $\beta_i \leq \alpha_i$.

We characterize the least-cost contract that implements work by all the agents as a unique Nash equilibrium of the induced game (i.e. the optimal unique implementation contract).[8] We require schemes to induce such a unique equilibrium only once the individual rewards are increased by any positive amount.[9] More precisely, we characterize the reward vector v^* that solves the following optimization problem:

$$Min_{v\in\Re^n} \sum_{j\in N} v_j \tag{3}$$

s.t. for all $\epsilon > 0$, we have:

for all $i \in N$, full effort $\mathbf{1}^n$ is a Nash equilibrium:

$$U_i(\mathbf{1}^n, v+\epsilon) > U_i(\mathbf{1}^n_{e_i=0}, v+\epsilon), \tag{IC}$$

where $\mathbf{1}^n_{e_i=0}$ is a vector of dimension n with all components being 1 except the ith component which is set to 0, and, there is no other Nash equilibrium, that is, for all $e \neq \mathbf{1}^n$, $\exists i \in N$ such that $e_i = a$, $a \in \{0,1\}$ and:

$$U_i(e_{e_i=1-a}, v+\epsilon) > U_i(e, v+\epsilon), \tag{UC}$$

where $e_{e_i=1-a}$ is identical to vector e except that the ith component is set to $1-a$ and $v+\epsilon = (v_1+\epsilon, ..., v_n+\epsilon)$.

The set of constraints (IC) ensures that no agent has an incentive not to exert effort when all the other agents exert effort (i.e. it is a Nash equilibrium of the effort game) and the set of constraints (UC) ensures that for each other outcome, at least one agent has an incentive not to exert effort (i.e. there is no other Nash equilibrium of the effort game).

3 The optimal reward scheme

The first step to characterize the optimal unique implementation contract is to show that it is characterized by the divide-and-conquer (DAC) property. The set of contracts exhibiting this property is obtained by ranking agents in an arbitrary fashion, and by providing each agent with a reward that would induce her to exert effort assuming that all the preceding agents in the ranking

[8]In Section 5 we investigate the case where the principal can costlessly select her most preferred equilibrium outcome. Thus, we we characterize the least-cost contract that ensures that the outcome where all agents work be one (of possibly many) equilibrium (partial implementation), as is done in Segal (1999) for instance.

[9]This solution concept is also used in Winter (2004) and Halac et al. (2020).

(the higher ranked agents) also exert effort and all lower ranked agents shirk. Intuitively, lower ranked agents are induced to exert effort by the others' choice to do so and can be offered smaller rewards.

So we first consider an arbitrary ranking of the set of agents, and we provide a preliminary result:

Lemma 1: *The reward scheme v^* is such that the agents are ordered, without loss of generality, from 1 to n, and such that agent i is indifferent between exerting effort and shirking when the $i-1$ previous agents also exert effort while the remaining $n-i$ agents do not exert effort.*

Notice that the proof of this result is not straightforward. It is in fact obtained through the complete characterization of the optimal scheme (see the Proof of Proposition 2). However, for the ease of exposition, we choose to present it as a preliminary result. This Lemma states that the most stringent constraint is the uniqueness constraint (UC) and not the Nash equilibrium implementation constraint (IC). Intuitively, the principal can induce agent i to exert effort no matter what the subsequent $i+1, ..., n$ agents do if she can induce agent i to exert effort when these agents do not do so. This type of incentive scheme is called a divide and conquer scheme in the literature.[10]

Let us introduce some notations. Let π_i^j be agent i's material payoff when the first j agents exert effort while the remaining $n-j$ agents shirk. Let us denote A_i^j and B_i^j the resulting level of disadvantageous (respectively, advantageous) inequality when the first j agents exert effort while the remaining agents do not:

$$A_i^j = \frac{1}{n-1}\sum_{k \neq i} \max\{\pi_k^j - \pi_i^j, 0\} \text{ and } B_i^j = \frac{1}{n-1}\sum_{k \neq i} \max\{\pi_i^j - \pi_k^j, 0\}. \tag{4}$$

Finally, let agent i's total disutility resulting from inequality be denoted as:

$$D_i^j = \alpha_i A_i^j + \beta_i B_i^j \tag{5}$$

The indifference condition described in Lemma 1 can be written as follows, considering agent i:

$$\pi_i^i - c = \Delta D_i^i, \tag{6}$$

where $\Delta D_i^i = D_i^i - D_i^{i-1}$.

Notice that, if agent i does not exhibit aversion to inequalities ($\alpha_i = \beta_i = 0$), then his material payoff is equal to his opportunity cost, $\pi_i^i = c$. The differential ΔD_i^i characterizes the impact of inequality aversion on agent i's payoff.

We can show the following result:

Proposition 1: *The reward scheme v^* is such that each agent i's disutility from inequality is larger*

[10] See Segal (2003) on contracts with externalities, or Rasmusen et al. (1991) on exclusionary contractual clauses.

if she exerts effort rather than if she shirks if the first $i-1$ agents exert effort and the remaining $n-i$ agents shirk, that is $\pi_i^i - c = \Delta D_i^i \geq 0$. As a consequence, an agent obtains a larger payoff if she is averse to inequality rather than selfish.

This result is rather counter intuitive. Indeed, a first intuition suggests that inequality aversion may cause a reduction in pay inequalities within organizations, which may be associated with smaller monetary performance incentives (see Cohn et al. 2014). In our model, an initial intuition suggests that the principal, in order to minimize the rewards, might have incentives to make the material payoffs smaller than the opportunity cost. However, Proposition 1 states that this is not possible because of the uniqueness constraint.

To understand the mechanism further, the proof uses the following argument. In order to induce the first agent to exert effort while all the other agents shirk, the principal has to provide him with a reward that is at least equal to the opportunity cost, that is $\pi_1^1 \geq c$. For the other agents, it is actually not possible that the result holds for the first $i-1$ agents and not for agent i. Indeed, if the first $i-1$ agents obtain a reward that is larger than c when the agents who precede them exert effort, their payoff increases when more agents exert effort. Hence, when the agents who precede agent i exert effort, if agent i receives a reward that is lower than the opportunity cost, she obtains the lowest payoff, whatever her decision. In this case, only disadvantageous inequality aversion plays a role ($B_i^i = B_i^{i-1} = 0$) and agent i is more disadvantaged when she exerts effort rather than when she shirks ($A_i^i > A_i^{i-1}$). We conclude that agent i's payoff must satisfy $\pi_i^i \geq c$. Hence, agent i's reward is $v_i^* = c - (i-1)w + \Delta D_i^i$, where $\Delta D_i^i \geq 0$.

We now proceed with the next step of the characterization by providing the following property:

Lemma 2: *The reward scheme v^* is such that $v_i^* - v_{i+1}^* = \pi_i^j - \pi_{i+1}^j > 0$ for all i and for $j = 1, ..., n$.*

This result is directly related to the result provided in Proposition 1. Indeed, Proposition 1 states that the first agent has to obtain a reward that is equal to the opportunity cost, $v_1^* = c$. According to Proposition 1, agent 2's reward is such that $v_2^* = c - w + \Delta D_2^2$ which does not depend on any of agent j's reward v_j when $j > 2$. Hence, the principal has an incentive to disadvantage agent 2 compared to agent 1 in order to minimize the total reward. The same logic applies for all subsequent agents.

Moreover, we can use this result in order to derive a simpler expression characterizing agent i's optimal payoff. Let $\Delta A_i^i = A_i^i - A_i^{i-1}$ and $\Delta B_i^i = B_i^i - B_i^{i-1}$ be the difference in disadvantageous (respectively, advantageous) inequality. Condition (6) can be rewritten as follows:

$$\pi_i^i - c = \alpha_i \underbrace{\frac{1}{n-1} \sum_{k<i} \left[(\pi_k^i - \pi_i^i) - (\pi_k^{i-1} - c) \right]}_{\Delta A_i^i} + \beta_i \underbrace{\frac{1}{n-1} \sum_{k>i} \left[\pi_i^i - c \right]}_{\Delta B_i^i}. \tag{7}$$

If the first $i-1$ agents exert efforts, agent i is disadvantaged compared to these agents, whether he exerts effort or not. He is also advantaged compared to the $n-i$ remaining agents who do not exert effort, but only if she does exert effort (otherwise she gets the same opportunity cost c than these agents). Notice that, according to Proposition 1, which holds for $\alpha_i \geq \beta_i \geq 0$, the difference in disadvantageous (respectively, advantageous) inequality has to be non negative, that is $\Delta A_i^i \geq 0$ and $\Delta B_i^i \geq 0$. The optimal reward scheme is such that disadvantageous inequality and advantageous inequality increase when agent i exerts effort rather than shirks.

Solving for the set of conditions (7), we obtain a characterization of the solution to the main problem:

Proposition 2 [Reward scheme]: *If the agents are averse to inequality, the optimal reward scheme that implements work as a unique Nash equilibrium of the induced game is such that:*

$$v_i^* = c - (i-1)w + \Delta D_i^i, \tag{8}$$

where

$$\Delta D_i^i = \frac{(i-1)\alpha_i}{n-1+(i-1)\alpha_i-(n-i)\beta_i}w, \tag{9}$$

for all $1 \leq i \leq n$.

This result characterizes the additional material payoff that the agents obtain because they exhibit aversion to inequality. It highlights that the stronger the aversion to inequality (i.e. the larger α_i or β_i), the larger the agent's reward. This static comparative result follows directly from condition (7) and from the fact that disadvantageous inequality and advantageous inequality increase when agent i exert effort rather than when she shirks. When an agent is more averse to inequality, these increases in inequality have to be compensated by an increase in the agent's reward.

It is also interesting to notice that disadvantageous inequality aversion is of first-order importance compared to advantageous inequality aversion. Indeed, using a first order approximation of agent i's reward around $(\alpha_i, \beta_i) = (0, 0)$, we have $v_i \sim c - (i-1)w + (i-1)\alpha_i w$ which depends on α_i but not on β_i. This is due to the fact that, when the higher ranked agents exert effort, an agent is only advantaged compared to the lower ranked agents who do not exert effort and who get the opportunity cost c. The disutility that is due to advantageous inequality aversion is thus proportional to the increase in the agent's material payoff when she decides to exert effort instead of shirking (still when only the previous agents do exert effort), as can be seen in condition (7). The DAC nature of the optimal contract awards a lot of importance to higher ranked agents, and as such to disadvantageous inequality-aversion parameters.

The optimal contract satisfies the DAC property with the optimal ranking, that is, the ranking that minimizes the principal's aggregate cost of providing incentives to exert effort. As such, it is important to notice that Proposition 2 does not provide insights on the agents' optimal ranking.

We cannot provide a full characterization of the optimal ranking as heterogeneity is bi-dimensional, each agent i being characterized by a pair of inequality aversion parameters, (α_i, β_i). However, we can characterize the optimal ranking when the agents are identical with respect to one dimension and heterogeneous with respect to the other one. To get some intuition, notice that in condition (7), the disadvantageous (advantageous) inequality difference is proportional to the number of agents who obtain a larger (lower) reward, that is:

$$\Delta A_i^i = \frac{1}{n-1}\left[w - (\pi_i^i - c)\right] i \text{ and } \Delta B_i^i = \frac{1}{n-1}\left[\pi_i^i - c\right](n-i). \tag{10}$$

Let us first consider the case where the agents have different disadvantageous inequality aversion parameters:

Proposition 3 [Disadvantageous inequality]: *Assume $\beta_j = \beta$ for all j. (i) If the agents are weakly averse to disadvantageous inequality ($\alpha_j < 1$ for all j), then the optimal ranking is such that an agent's rank decreases as her disadvantageous inequality parameter is lower ($\alpha_1 \geq \alpha_2 \geq ... \geq \alpha_n$). (ii) If the agents are strongly averse to disadvantageous inequality ($\alpha_j > 1$ for all j), then the optimal ranking is such that the agents' rank is a U-shaped function of the disadvantageous inequality parameters ($\exists 1 < k < n$ such that $\alpha_1 \geq ... \geq \alpha_k$ and $\alpha_k \leq ... \leq \alpha_n$).*

If the agents give more weight to disadvantageous inequality than to their own monetary payoff (case (i)), the most averse agents lie at the top of the reward distribution. However, if the agents give more weight to their own payoff than to disadvantageous inequality (case (ii)), then the optimal ranking is non monotonic and the most inequality-averse agents lie at both ends of the reward distribution.

To get some intuition on this result, it is sufficient to focus on the disutility resulting from disadvantageous inequality. Assume that agent i is ranked at position j. Her disutility resulting from disadvantageous inequality is $\alpha_i \Delta A_j^j = \frac{\alpha_i}{n-1}\left[w - (\pi_j^j - c)\right] j$. A marginal increase in α_i leads to a marginal disutility level $\Delta A_j^j + \alpha_i \frac{\partial \Delta A_j^j}{\partial \alpha_i}$. The first term increases, while the second term decreases, when the rank j increases. When the degree of aversion to disadvantageous inequality is sufficiently low ($\alpha_i < 1$), the first term dominates, and then the principal has incentives to first rank the most inequality-averse agents. When the degree of aversion to disadvantageous inequality is sufficiently large ($\alpha_i > 1$), the second term is sufficiently strong and the optimal ranking is non monotonic. In other words, the agents' ranking is characterized by the magnitude of the disutility resulting from disadvantageous inequalities. An agent's disadvantageous inequality-aversion parameter has both a direct effect on this disutility and an indirect effect resulting from its impact on this agent's optimal bonus. The direct effect is positive, while the indirect effect is negative, and the net impact depends on the fundamentals.

We now characterize the optimal ranking when the agents only differ in terms of their aversion to advantageous inequality:

Proposition 4 [Advantageous inequality]: *Assume $\alpha_j = \alpha$ for all j. The optimal ranking is such that the rank is a U-shaped function of the advantageous inequality parameters ($\exists 1 < k < n$ such that $\beta_1 \geq ... \geq \beta_k$ and $\beta_k \leq ... \leq \beta_n$).*

The optimal ranking of heterogeneous agents who are averse to advantageous inequality is non monotonic: The most inequality-averse agents lie at both ends of the reward distribution. When agents are heterogeneous in their advantageous inequality-aversion parameters only, the agents' ranking is characterized by the magnitude of the disutility resulting from advantageous inequalities. An agent's disadvantageous inequality-aversion parameter has both a direct effect on this disutility and an indirect effect resulting from its impact on this agent's optimal bonus. As in the previous case, there is a trade-off between direct and indirect effects, but this time the trade-off has always bite: there is a U-shape relationship.

4 Payoffs and inequality

The characterization provided in Proposition 2 can be used to provide some insights about the effects driven by social comparisons compared to the case where agents do not exhibit social preferences. We obtain the following result:

Proposition 5 [Effect of inequality aversion]: *Under the optimal unique implementation contract, we have the following conclusions:*
(i) The material payoff of an agent is larger when he is inequality averse (i.e. when $\beta_i \geq 0$ and $\alpha_i \geq 0$ instead of $\alpha_i = \beta_i = 0$).
Assuming that the agents are symmetric ($\alpha_k = \alpha$ and $\beta_k = \beta$ for all k), we also have that:
(ii) The magnitude of the difference between any two subsequent agents' material payoffs, $|\pi_i - \pi_{i+1}|$, is lower when the agents are averse to inequality (i.e. when $\beta \geq 0$ and $\alpha \geq 0$ instead of $\alpha = \beta = 0$)

These properties mostly follow from Proposition 2. For instance, regarding point (i), the optimal reward scheme is such that disadvantageous inequality and advantageous inequality increase when agent i exerts effort rather than when she shirks. These increases in inequality levels have to be compensated by an increase in the agent's reward when she is averse to inequality.

We can also use the characterization of the optimal reward scheme to highlight some non-intuitive effects of social comparisons on inequality levels. We obtain:

Proposition 6 [Effect on inequality]: *Under the optimal unique implementation contract, we have the following conclusions:*

(i) A marginal increase in disadvantageous inequality aversion may lead to an increase in inequality at the bottom of the rewards distribution. Formally, $\frac{\partial|\pi_i - \pi_{i+1}|}{\partial \alpha} \geq 0$ if and only if $i \geq 2$ and $\alpha \geq \sqrt{\frac{n-1-(n-i)\beta}{i(i-1)}}$.

(ii) A marginal increase in advantageous inequality aversion may lead to an increase in inequality at the bottom of the rewards distribution. Formally, $\frac{\partial|\pi_i-\pi_{i+1}|}{\partial\beta} \geq 0$ *if and only if* $i = n-1$ *or* $\beta \in \left[1-\sqrt{\frac{i(i-1)}{(n-i)(n-i-1)}}(1+\alpha), 1\right[$.

These effects are direct implications from the characterization of optimal rewards provided in Proposition 2. The two cases highlight that an increase in the intensity of aversion to inequalities may actually result in higher inequality through the impact of both types of inequality aversion on the agents' rewards. Yet, these results also highlight that such effect differs depending on whether the focus is on disadvantageous inequalities or on advantageous inequalities.

5 (How) does coordination matter?

In the previous sections, we have highlighted several qualitative properties of the optimal unique implementation contract. As such, we characterized the optimal reward scheme that solves any potential coordination problem. This raises the question about whether the coordination problem does actually matter. Does the existence of such a problem drastically affect the characterization of the optimal contract, or does it have little effect on it? To answer this question, we now solve for the optimal partial implementation contract, that is, the least-cost contract inducing all agents to exert effort as one of the equilibria of the induced game. We obtain the following characterization:

Proposition 7: *If the agents are averse to inequality, the optimal reward scheme that implements work as a Nash equilibrium of the induced game is such that:*

$$v_i^* = c - (n-1)w - J_i w, \tag{11}$$

where

$$J_1 = \frac{\beta_1}{1-\beta_1} \tag{12}$$

and

$$J_i = \frac{\beta_i + \frac{\alpha_i+\beta_i}{n-1}\left(\sum_{j=1}^{i-1} J_j\right)}{1-\beta_i+\frac{i-1}{n-1}(\alpha_i+\beta_i)}, \tag{13}$$

for all $2 \leq i \leq n$.

Proposition 7 allows to conclude that the coordination problem does matter as it deeply affects the characterization of the optimal contract. Indeed, using Propositions 2 and 7, we conclude that optimal unique and partial implementation contracts drastically differ. Under partial implementation, social comparisons negatively affect monetary incentives: compared to the case of selfish agents, the reward scheme provides all agents with smaller payments. This conclusion is entirely reversed when dealing with unique implementation: all agents are provided with higher payments (compared to the case where they do not exhibit social preferences).

It is also interesting to notice that aversion to advantageous inequality is of first-order importance compared to disadvantageous inequality aversion. Indeed, using a first-order approximation of agent i's reward around $(\alpha_i, \beta_i) = (0, 0)$, we have $v_i \sim c - (n-1)w - \beta_i w$ which depends on β_i but not on α_i. This qualitative result is also entirely reversed when dealing with unique implementation: disadvantageous inequality aversion is of first order importance in this case.
Intuitively, the fact that cooperation be only one of several equilibria allows the principal to reduce the overall cost of the contractual scheme by relaxing the incentive constraints for all agents: this actually implies that advantageous inequality-aversion parameters then become the most relevant parameters. This has to be contrasted with the case of optimal unique implementation, where the DAC structure of the contract tends to award more importance to higher ranked agents, and thus to aversion to disadvantageous inequalities.

We now rely on this characterization to highlight another notable effect of the coordination problem: namely, the induced differences in terms of the agents' optimal ranking. Since there is a fundamental difference with the case of optimal unique implementation in terms of the characterization of the optimal reward scheme, The same type of qualitative differences is expected for the next results. In order to understand the effect of each fundamental, we proceed with the analysis in several steps. First, we assume that agents share the same degree of aversion to advantageous inequalities. We then consider the case where they share the same degree of aversion to disadvantageous inequalities. Finally, we provide some partial insights about the general case. We have the first following result:

Proposition 8 [Disadvantageous inequality]: *Assume $\beta_j = \beta$ for all j. Then aversion to disadvantageous inequality has no effect on the agents' ranking: $v_i = v \ \forall i \in I$.*

This important feature is perfectly illustrated by Proposition 8: Aversion to advantageous inequalities has a first-order effect on the characterization of the optimal ranking induced by the partial implementation contract. We now analyze the polar case where the agents' degree of aversion of disadvantageous inequalities is the same. We obtain the following results:

Proposition 9 [Advantageous inequality]: *Assume $\alpha_j = \alpha$ for all j. The optimal ranking is increasing in the magnitude of aversion to advantageous inequality: $\beta_1 \leq ... \leq \beta_n$.*

This case highlights the asymmetric effect of social comparisons. Aversion to advantageous inequality has a first-order effect on the optimal ranking, as it flattens the distribution of payments when agents are homogeneous with respect to this fundamental. Aversion to disadvantageous inequality does not have a similar type of effect: when agents are homogeneous with respect to this fundamental, the optimal ranking is characterized by increasing degrees of aversion to advantageous inequality for lower-ranked agents.

We now move on to the general case, and we specifically highlight the potential non-monotonicity of the optimal ranking. We obtain:

Proposition 10 [General case]: *The optimal ranking satisfies the following conditions:* $\beta_1 \leq \beta_2$ *and, for any* $i \geq 3$*:*

$$(\beta_i - \beta_{i-1}) + \frac{\sum_{l=1}^{i-2} I_l}{n-1} \left[(\alpha_i + \beta_i) - (\alpha_{i-1} + \beta_{i-1}) - \beta_{i-1}\alpha_i + \beta_i \alpha_{i-1} \right] + \frac{i-2}{n-1} \left[\beta_i \alpha_{i-1} - \beta_{i-1} \alpha_i \right] \geq 0 \tag{14}$$

As such, if agents' aversion to inequalities parameters are such that there exists a ranking satisfying $\frac{\beta_i}{\beta_{i-1}} > \frac{\alpha_i}{\alpha_{i-1}} \geq 1$ *for any* $i \geq 3$ *then it is the optimal ranking, and it satisfies* $\alpha_i \geq \alpha_{i-1}$ *and* $\beta_i > \beta_{i-1}$ *for any* $i \geq 3$*. When these conditions are not satisfied, the optimal ranking may satisfy* $\alpha_i \leq \alpha_{i-1}$ *for some* $i \in I$*.*

The fact that it is not possible to obtain a closed-form characterization was fairly expected as heterogeneity is again bi-dimensional. Nonetheless, Proposition 10 highlights a sufficient condition ensuring that the optimal ranking satisfies some particular form of monotonicity. When this condition is not satisfied, it is not possible to obtain a clear-cut conclusion and the optimal ranking may be non-monotonic.

6 Conclusion

While social comparisons seem to strongly affect workers well-being and performance, little is known about how organizations should account for these behaviors. This paper theoretically analyzes the interplay between agents' coordination problem and inequality aversion. We characterize the specific features of the optimal contract when multiple agents interact within an organization and are averse to inequalities.

We show that inequality aversion may result in higher inequality levels within the organization, and that it should be associated with larger monetary incentives. This is an important qualitative feature. We further highlight that disadvantageous inequality aversion is of first-order importance compared to advantageous inequality aversion. This is consistent with conclusions from existing empirical studies, which highlight the asymetric importance of the different types of inequality aversion. The distribution of rewards may also be non monotonic, and the most inequality-averse agents may lie at both ends of the reward distribution. We then conclude by discussing the changes that result when assuming that the principal could select her preferred equilibrium outcome at no cost.

These results provide a first step in the analysis of optimal contract design in situations where agents exhibit social preferences (here, aversion to inequalities). It opens up many avenues for future research: for instance, it would be very interesting to consider situations characterized by different types of agents' interactions, or to analyze settings characterized by repeated interactions.

Appendix

Proof of Lemma 1: See the proof of Proposition 2.□

Proof of Proposition 1:

If $i = 1$, we have

$$\pi_1^1 - c = D_1^1 - D_1^0 = D_1^1 = \alpha_1 \max\{c - \pi_1^1, 0\} + \beta_1 \max\{\pi_1^1 - c, 0\}. \tag{15}$$

Hence, we must have $\pi_1^1 - c \geq 0$.

Now, assume that $\pi_k^k \geq c$ for all $k < i$ while $\pi_i^i < c$. Hence, we have $\pi_i^i < c = \pi_j^i \leq \pi_k^i$ for $k < i$ and $j > i$, and $\pi_i^{i-1} = c = \pi_j^{i-1} \leq \pi_k^k \leq \pi_k^{i-1}$. Hence, in this case $B_i^i = B_i^{i-1} = 0$ and:

$$A_i^i - A_i^{i-1} = \frac{1}{n-1}\sum_{k<i}\Big[(\pi_k^i - \pi_i^i) - \max\{\pi_k^i - w - c, 0\}\Big] + \frac{1}{n-1}\sum_{k>i}\Big[(c - \pi_i^i) - 0\Big] > 0. \tag{16}$$

Thus is sufficient to prove the result.□

Proof of Lemma 2: See Step 2 in the Proof of Proposition 2.□

Proof of Proposition 2:

Notice that the utility function of agent i can be rewritten as follows:

$$U_i(e,v) = (1-\beta_i)\pi_i(e,v) + \frac{\beta_i}{n-1}\sum_{k\neq i}\pi_k(e,v) - \frac{\alpha_i+\beta_i}{n-1}\sum_{k\neq i}\max\{\pi_k(e,v) - \pi_i(e,v), 0\}. \tag{17}$$

The divide and conquer property holds if, for a given ordering of the agents $1, ..., n$, each agent prefers to exert effort when all the preceding agents exert effort and the subsequent agents do not. Using (17), we can show that this condition holds for agent $i = 1$ if and only if $v_1 \geq c$ and for $i \geq 2$ if and only if:

$$\begin{aligned}(1-\beta_i)\left[v_i + (i-1)w - c\right] + (i-1)\frac{\beta_i}{n-1}w \geq \\ \frac{\alpha_i+\beta_i}{n-1}\left[\sum_{k<i}\left(\max\{v_k - v_i, 0\} - \max\{v_k + (i-2)w - c, 0\}\right)\right] \\ + \frac{\alpha_i+\beta_i}{n-1}(n-i)\max\{c - v_i - (i-1)w, 0\}\end{aligned} \tag{18}$$

Step 1: We show that, the least cost contract that is characterized by the divide and conquer property is such that (18) is binding for all i.

Assume that there exists an agent i such that (18) is not binding. This implies that we have to solve a problem with $n-1$ inequalities and n unknowns. Thus, there exists an agent l such that $v_l = -\infty$. We can thus rewrite condition (18) for this agent as follows:

$$\left(1 - \beta_l + \frac{n-l}{n-1}\left(\alpha_l + \beta_l\right)\right)\left(v_l + (l-1)w - c\right) + \frac{l-1}{n-1}\left(\alpha_l + \beta_l\right) v_l \geq \\ \frac{\alpha_l + \beta_l}{n-1} \sum_{k<i} \left[\left(v_k - \max\{v_k + (l-2)w - c, 0\}\right) - \frac{\beta_l}{n-1} w\right] \quad (19)$$

This implies that there exists an agent $l' < l$ such that $v_{l'} = -\infty$. Repeating this reasoning, we can conclude that $v_1 = -\infty$, which is a contradiction since $v_1 \geq c$. We conclude that the least cost DAC contract is characterized by $v_1 = c$ and:

$$\left(1 - \beta_i\right)\left[v_i + (i-1)w - c\right] + (i-1)\frac{\beta_i}{n-1} w = \\ \frac{\alpha_i + \beta_i}{n-1}\left[\sum_{k<i}\left(\max\{v_k - v_i, 0\} - \max\{v_k + (i-2)w - c, 0\}\right)\right] \\ + \frac{\alpha_i + \beta_i}{n-1}(n-i)\max\{c - v_i - (i-1)w, 0\}, \quad (20)$$

for all $i \geq 2$.

Step 2: We show that, the least cost contract that is characterized by the divide and conquer property is such that $v_i = c - (i-1)w + \Delta D_i^i$ for all i.

Using the result from Proposition 1, after some computations, condition (20) can be rewritten as follows:

$$\left[1 - \beta_i + \left(\alpha_i + \beta_i\right)\frac{i-1}{n-1}\right]\left(v_i + (i-1)w - c\right) = \frac{\alpha_i + \beta_i}{n-1}\left[\sum_{k<i}\max\{v_i - v_k, 0\}\right] + \alpha_i \frac{i-1}{n-1} w \quad (21)$$

Thus, in order to minimize the cost of the scheme, the principal chooses to rank the agents such that $v_i \leq v_k$ for all $k < i$ and $v_i = c - (i-1)w + \Delta D_i^i$ for all i. Notice that $v_i - v_{i-1} = \left(I_i - I_{i-1} - 1\right) w$, which is non positive if and only if $I_i - I_{i-1} \leq 1$, which is always true since $I_k \leq 1$ for all k.

Step 3: We show that the reward scheme described in the Proposition is such that the situation where all the agents exert effort is a Nash equilibrium of the effort choice game.

The reward scheme in the Proposition has the DAC property. Hence, agent n has no incentive to deviate from the situation where all the agents exert. Let us consider agent $i < n$. Let us denote π_k^n the material payoff of agent k when all the agents exert effort. We have $\pi_k^n = c + (n-k)w + I_k w$.

The difference between two successive terms is $\pi_{k+1}^n - \pi_k^n = (I_{k+1} - I_k - 1)\, w \leq 0$. When agent i deviates from the situation where all the agents exert effort, the material payoff of agent k and $k \neq i$ is $\pi_k^n - w$. This payoff is larger than c. Indeed, we have $\pi_k^n - w - c = (n-k-1)w + I_k w \geq 0$ if and only if $k < n$.

Using these remarks, we have that agent $i < n$ has no incentives to deviate from the situation where all the agents exert effort if and only if:

$$(1-\beta_i)\,(\pi_i^n - c) + \beta_i w + \frac{\alpha_i + \beta_i}{n-1}(i-1)\,(\pi_i^n - w - c) + \frac{\alpha_i + \beta_i}{n-1}\sum_{i<k<n}(\pi_k^n - w - c) \geq 0 \tag{22}$$

This concludes the proof of Step 3.

Step 4: We show that the least cost DAC contract is such that the situation in which all the agents exert effort is the unique Nash equilibrium of the effort choice game.

In the previous steps we have characterized the least cost DAC contract and we have shown that it implements work as a Nash equilibrium. This contract is such that any situation where the i first agents ($0 \leq i < n$) exert effort is not a Nash equilibrium. It remains to show that the remaining possible outcomes are not Nash equilibria.

Let us show that when the least cost DAC contract is implemented, agent i has an incentive to exert effort as long as $i-1$ other agents exert effort and the remaining agents do not. Let P_{i-1} denote the set composed of the $i-1$ other agents who exert effort. The material payoff of an agent k who exerts effort when a total of i agents exert effort is $\pi_k^i = c + (i-k)w + I_k w$. The material payoff of agent i is $\pi_i^i = c + \Delta D_i^i \geq c$.

When all agents $k \in P_{i-1}$ exert effort and all the remaining agents do not exert effort, agent i has no incentives to deviate if and only if:

$$(1-\beta_i)\left(\pi_i^i - c\right) + \frac{i-1}{n-1}\beta_i \geq \frac{\alpha_i + \beta_i}{n-1}\left[\sum_{k\in P_{i-1}}\Big(\max\{\pi_k^i - \pi_i^i, 0\} - \max\{\pi_k^i - w - c, 0\}\Big)\right], \tag{23}$$

or,

$$(1-\beta_i)I_i w + \frac{i-1}{n-1}\beta_i w \geq \frac{\alpha_i + \beta_i}{n-1}\left[\sum_{k\in P_{i-1}}\left(\max\{(i-k+I_k-I_i)w, 0\} - \max\{(i-k+I_k-1)w, 0\}\right)\right]. \tag{24}$$

Let $\underline{P}_i$ be the set of agents $k < i$. Notice that $-1 \leq I_k - I_i \leq 1$ and $-1 \leq I_k - 1 \leq 0$. Hence, condition (24) can be rewritten as follows:

$$(1-\beta_i)I_i w + \frac{i-1}{n-1}\beta_i \geq \frac{\alpha_i+\beta_i}{n-1}\left[\sum_{k\in P_{i-1}\cap \underline{P}_i} \max\{(i-k+I_k-I_i)w, 0\}\right]. \tag{25}$$

The term in brackets in the right hand side is maximum when the set P_{i-1} is only composed of agents k with $k<i$. We know that condition (25) holds in this case (by definition of the DAC property).

This is sufficient to conclude the proof of Step 4.

Step 5: We show that the least cost unique implementation contract is the least cost DAC unique implementation contract.

A unique implementation contract is such that, for any outcome $e \neq 1^n$, at least one agent has an incentive to deviate. Consider outcome $(0, ..., 0)$, that is, no agent does exert effort. In this case, at least one agent has an incentive to deviate and to change her choice from not exerting effort to exerting effort (i.e. the DAC property holds for this agent). Let us rank this agent first. Consider the outcome in which only the first agent exerts effort, $(1, 0, ..., 0)$. In this case, at least one agent who does not exert effort has an incentive to deviate (i.e. the DAC property holds for this agent). Let us rank this agent second. Consider outcome $(1, 1, 0..., 0)$, that is, only the two first agent exert effort. There are two possibilities here: (i) one agent who does not exert effort has an incentive to deviate (i.e. the DAC property holds for this agent) or the first agent has an incentive to deviate. Assume that the latter holds, that is, agent 1 prefers $(0, 1, 0..., 0)$ over $(1, 1, 0, ..., 0)$, then:

$$(1-\beta_1)(c-v_1-w) - \frac{\beta_1}{n-1}w - \frac{\alpha_1+\beta_1}{n-1}\left(\max\{v_2-c, 0\} - \max\{v_2-v_1, 0\}\right) \geq 0. \tag{26}$$

We know that the DAC holds for agent 1 and that it is equivalent to $v_1 \geq c$. Thus, the left hand side in condition (26) is negative, which is a contradiction. Hence, at least one agent who does not exert effort has an incentive to deviate. Let us rank this agent in third position.

Let us show that this result holds for any outcome such that the first j agents exert effort and the DAC property holds for these agents. Using the same argument as in Step 1, we can show that the rewards of these agents are the rewards they receive under the least cost DAC unique implementation contract. The payoff of these agents, when the j first agents exert effort, is denoted π_k^j with $k = 1, ..., j$ and it is thus such that $\pi_k^j \geq c+w$ for all $k<j$. Consider the outcome where the first j agents exert effort and the other agents do not exert effort. Agent $i<j$ has an incentive to deviate if and only if:

$$(1-\beta_i)\left(c-\pi_i^j\right) - \frac{\beta_i}{n-1}(j-1)w - \frac{\alpha_1+\beta_1}{n-1}\left(\sum_{k\leq j, k\neq i}\left[\max\{\pi_k^j - w - c, 0\} - \max\{\pi_k^j - \pi_i^j, 0\}\right]\right) \geq 0. \tag{27}$$

We have $\pi_i^j \geq c+w$ and then the left hand side in condition (27) is negative, which is a contradiction. We consider a unique implementation contract here, hence at least one agent who does not exert effort has an incentive to deviate. Let us rank this agent in position $j+1$. This concludes the proof of our claim and of step 5.

Step 6: It remains to show that adding an arbitrarily small amount payment to each reward makes the incentive scheme a unique implementation scheme.

One can show that the scheme characterized in Proposition 2 satisfies this definition. Consider that agent i's reward is $v_i^* + \epsilon$, where $\epsilon > 0$ is arbitrarily small. First, let us show that each agent (strictly) prefers to exert effort when all the preceding agents exert effort and the subsequent agents do not. For agent $i = 1$, we have $v_1^* + \epsilon = c + \epsilon > c$. For $i \geq 2$, the needed condition is:

$$(1-\beta_i)\left[v_i^* + \epsilon + (i-1)w - c\right] + (i-1)\frac{\beta_i}{n-1}w > \frac{\alpha_i+\beta_i}{n-1}\left[\sum_{k<i}\left(\max\{v_k^* - v_i^*, 0\} - \max\{v_k^* + \epsilon + (i-2)w - c, 0\}\right)\right] + \frac{\alpha_i+\beta_i}{n-1}(n-i)\max\{c - v_i^* - \epsilon - (i-1)w, 0\} \quad (28)$$

First, notice that $v_k^* + \epsilon + (i-2)w - c = (I_k - 1)w + \epsilon < 0$ because $I_k - 1 < 0$ and ϵ is arbitrarily small. Second, notice that $c - v_i^* - \epsilon - (i-1)w = -I_i - \epsilon < 0$. Hence, condition (28) is equivalent to $(1-\beta_i)\epsilon > 0$, which is true. Hence, for each outcome except the outcome at which all the agents exert effort, at least one agent strictly prefers to deviate.

Moreover, the outcome at which all the agent exert effort is a (strict) Nash equilibrium. It is sufficient to notice that condition (22) holds with a strict inequality when adding ϵ to each reward.□

Proof of Proposition 3: To prove the result, we can focus on the function $I(i,\alpha) \equiv \frac{(i-1)\alpha}{n-1+(i-1)\alpha-(n-i)\beta}$ and its cross derivative with respect to i and α. We can easily show that the cross derivative has the same sign as $n-1+\alpha-n\beta-i(\alpha-\beta)$. This expression is a linear function that decreases when i increases and its value is $(n-1)(1-\beta) > 0$ when $i = 1$ and $(n-1)(1-\alpha)$ when $i = n$. Hence, if $\alpha < 1$, the cross derivative is always positive. If $\alpha \geq 1$, the cross derivative is positive when i is below a threshold $1 < k < n$ and negative when i is above this threshold.□

Proof of Proposition 4: To prove the result, we can focus on the function $I(i,\beta) \equiv \frac{(i-1)\alpha}{n-1+(i-1)\alpha-(n-i)\beta}$ and its cross derivative with respect to i and β. We can easily show that the cross derivative has the same sign as $1+\alpha+(1-\beta)n-i(\alpha-\beta+2)$. This expression is a linear function that decreases when i increases and its value is $(n-1)(1-\beta) > 0$ when $i = 1$ and $-(1+\alpha)(n-1) < 0$ when $i = n$. Hence, the cross derivative is positive when i is below a threshold $1 < k < n$ and negative when i is above this threshold.□

Proof of Proposition 5: Proof of part (i) comes from the fact that $I_i \geq 0$. To prove part (ii), notice that we have $v_1 \geq v_2 \geq ... \geq v_n$ and $\pi_i - \pi_{i+1} = (1 + I_i - I_{i+1})\, w$. Moreover, when $\alpha_k = \beta_k = 0$ for $k = i, i+1$, we have $\pi_i - \pi_{i+1} = w$. Hence it is sufficient to show that $I_i - I_{i+1} \leq 0$. One can easily show that this is equivalent to $-(n-1) - (n-i)\beta + (i-1)\beta \leq 0$, which is always true because $\beta < 1$.

Proof of Proposition 6: Let us prove point (i). We have $\alpha_k = \alpha$ and $\beta_k = \beta$ for all k. We can show that:

$$\frac{\partial\, |\pi_i - \pi_{i+1}|}{\partial \alpha} = \frac{(n-1)(1-\beta)\left[i(i-1)\alpha^2 - (n-1-(n-i)\beta)\right]}{\left(n-1+(i-1)\alpha-(n-i)\beta\right)^2 \left(n-1+i\alpha-(n-i-1)\beta\right)^2} w, \tag{29}$$

which is positive if and only if $\alpha \geq \sqrt{\frac{n-1-(n-i)\beta}{i(i-1)}}$ and $i \geq 2$.

Now we prove point (ii). We have $\alpha_k = \alpha$ and $\beta_k = \beta$ for all k. We can show that:

$$\frac{\partial\, |\pi_i - \pi_{i+1}|}{\partial \beta} = \frac{-(n-i)\,(n-i-1)(n-1)\,(\beta^2 + 2\beta) + (i-1)(n-i)(n-1+i\alpha)^2 - i(n-i-1)(n-1+(i-1)\alpha)^2}{\left(n-1+(i-1)\alpha-(n-i)\beta\right)^2 \left(n-1+i\alpha-(n-i-1)\beta\right)^2} \alpha w, \tag{30}$$

which is positive if and only if $i = n-1$ or $\beta \in \left[1 - \sqrt{\frac{i(i-1)}{(n-i)(n-i-1)}}(1+\alpha), 1\right[$.

Proof of Proposition 7:

Notice that the utility function of agent i can be rewritten as follows:

$$U_i(e,v) = (1-\beta_i)\pi_i(e,v) + \frac{\beta_i}{n-1}\sum_{k \neq i} \pi_k(e,v) - \frac{\alpha_i + \beta_i}{n-1}\sum_{k \neq i} \max\{\pi_k(e,v) - \pi_i(e,v), 0\}, \tag{31}$$

The outcome at which all the agents exert effort is a Nash equilibrium of the induced effort choice game if and only if we have, for any $i \in I$:

$$(1-\beta_i)\left[v_i + (n-1)w - c\right] + \beta_i w - \frac{\alpha_i + \beta_i}{n-1}\sum_{k \neq i}\left[max\{v_k - v_i, 0\} - max\{v_k + (n-2)w - c, 0\}\right] \geq 0 \tag{32}$$

We claim that the least cost contract must satisfy $v_k + (n-2)w - c \leq 0$ for all $k \in I$. Assume that there is (at least) one agent for which this does not hold. Let us denote $\bar{N} \subset N$ such that $\forall k \in \bar{N}$ we have:

$$v_k + (n-2)w - c > 0$$

while any $j \notin \bar{N}$ is such that $v_j + (n-2)w - c \leq 0$ is satisfied. This implies that $\max_{j \notin \bar{N}} v_j < \min_{k \in \bar{N}} v_k$ is satisfied. Thus, ranking all agents in decreasing order with respect to their payment, if

$|\bar{N}| = m$ then subset $\bar{N}$ includes exactly the first m agents in this ranking $v_1 \geq ... \geq v_n$. Moreover, by definition: $\forall j \geq m+1$ we know that $v_j + (n-2)w - c \leq 0$ is satisfied.

Let us consider the first agent in the ranking. This agent's related participation constraint is:

$$(1-\beta_1)\left[v_1 + (n-1)w - c\right] + \beta_1 w + \frac{\alpha_1 + \beta_1}{n-1} \sum_{k=2}^{m} \left[v_k + (n-2)w - c\right] \geq 0 \tag{33}$$

This constraint is actually vacuous as v_1 satisfies $v_1 > c - (n-2)w$ by definition of $\bar{N}$, while the above inequality can be rewritten as

$$v_1 \geq c - (n-1)w - \frac{1}{1-\beta_1}\left[\beta_1 w + \frac{\alpha_1 + \beta_1}{n-1} \sum_{k=2}^{m} \left[v_k + (n-2)w - c\right]\right]$$

and it is easily checked that the following inequality is satisfied:

$$c - (n-2)w > c - (n-1)w - \frac{1}{1-\beta 1}\left[\beta_1 w + \frac{\alpha_1 + \beta_1}{n-1} \sum_{k=2}^{m} \left[v_k + (n-2)w - c\right]\right]$$

In other words, $v_1 + (n-2)w - c > 0$ is the relevant constraint: compared to the benchmark situation where $v_1 + (n-2)w - c \leq 0$ would hold, the payment of this agent has increased. Now, looking at the second agent in the ranking, his participation constraint is:

$$(1-\beta_2)\left[v_2 + (n-1)w - c\right] + \beta_2 w - \frac{\alpha_2 + \beta_2}{n-1}\left[v_1 - v_2\right] + \frac{\alpha_2 + \beta_2}{n-1} \sum_{k \in \bar{N}, k \neq 2} \left[v_k + (n-2)w - c\right] \geq 0 \tag{34}$$

or

$$(1-\beta_2)\left[v_2 + (n-1)w - c\right] + \beta_2 w + \frac{\alpha_2 + \beta_2}{n-1} \sum_{k \in \bar{N}, k \neq 1} \left[v_k + (n-2)w - c\right] \geq 0 \tag{35}$$

Finally, we can rewrite this inequality as:

$$v_2 \geq c - (n-1)w - \frac{1}{1-\beta_2}\left[\beta_2 w + \frac{\alpha_2 + \beta_2}{n-1} \sum_{k \in \bar{N}, k \neq 1} \left[v_k + (n-2)w - c\right]\right]$$

and it is easily checked that the following inequality is satisfied:

$$c - (n-2)w > c - (n-1)w - \frac{1}{1-\beta_2}\left[\beta_2 w + \frac{\alpha_2 + \beta_2}{n-1} \sum_{k \in \bar{N}, k \neq 1} \left[v_k + (n-2)w - c\right]\right]$$

In other words, $v_2 + (n-2)w - c > 0$ is the relevant constraint: compared to the benchmark situation where $v_2 + (n-2)w - c \leq 0$ would hold, the payment of this agent has also increased. A similar reasoning allows one to quickly conclude that the payments of all agents in $\bar{N}$ has increased compared to the benchmark situation.

Now, for agent $m+1$ the participation constraint is:

$$(1-\beta_{m+1})\left[v_{m+1}+(n-1)w-c\right]+\beta_{m+1}w-\frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[v_k-v_{m+1}\right]$$

$$+\frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[v_k+(n-2)w-c\right]\geq 0 \tag{36}$$

or

$$(1-\beta_{m+1})\left[v_{m+1}+(n-1)w-c\right]+\beta_{m+1}w+\frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[v_{m+1}+(n-2)w-c\right]\geq 0 \tag{37}$$

Finally, we can rewrite this inequality as:

$$(1-\beta_{m+1})\left[v_{m+1}+(n-1)w-c\right]+\beta_{m+1}w\geq\frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[c-(n-2)w-v_{m+1}\right] \tag{38}$$

while we would have, in the benchmark situation, the following inequality:

$$(1-\beta_{m+1})\left[v_{m+1}+(n-1)w-c\right]+\beta_{m+1}w\geq\frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[v_k-v_{m+1}\right] \tag{39}$$

Keep in mind that, in the benchmark situation, we have $v_k+(n-2)w-c\leq 0$ for any $k\in\bar{N}$, and we then easily check that:

$$\frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[v_k-v_{m+1}\right]-\frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[c-(n-2)w-v_{m+1}\right]=\sum_{k\in\bar{N}}\left[v_k+(n-2)w-c\right]$$

Since the final term is non-positive in the benchmark situation, this inequality implies that, compared to the benchmark situation, the payment to agent $m+1$ has at best remained the same.

Finally, for agent $m+2$, we obtain the following participation constraint:

$$(1-\beta_{m+2})\left[v_{m+2}+(n-1)w-c\right]+\beta_{m+2}w\geq\frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\sum_{k\in\bar{N}}\left[c-(n-2)w-v_{m+2}\right]$$

$$+\frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\left[v_{m+1}-v_{m+2}\right] \tag{40}$$

while we would have, in the benchmark situation, the following inequality:

$$(1-\beta_{m+2})\left[v_{m+2}+(n-1)w-c\right]+\beta_{m+2}w\geq\frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\sum_{k\in\bar{N}}\left[v_k-v_{m+2}\right]+\frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\left[v_{m+1}-v_{m+2}\right] \tag{41}$$

Again, in the benchmark situation, we have $v_k+(n-2)w-c\leq 0$ for any $k\in\bar{N}$, and we then easily check that:

$$\frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\sum_{k\in\bar{N}}\left[v_k-v_{m+2}\right]+\frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\left[v_{m+1}-v_{m+2}\right]$$

$$\leq\frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\sum_{k\in\bar{N}}\left[c-(n-2)w-v_{m+2}\right]+\frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\left[v_{m+1}-v_{m+2}\right] \tag{42}$$

This inequality implies that, compared to the benchmark situation, the payment to agent $m+2$ has at best remained the same. A similar reasoning applies to any agents who do not belong to $\bar{N}$. To summarize, compared to the benchmark situation, the payments to all agents in $\bar{N}$ have increased and those to non-members of $\bar{N}$ have at best remained the same. This contradicts the fact that this contract is least cost, and we conclude by contradiction that $v_k+(n-2)w-c\leq 0$ for all $k\in I$.

Now, the agent (say, agent 1) whose payment is the largest has to satisfy the following constraint:

$$(1-\beta_1)\left[v_1+(n-1)w-c\right]+\beta_1 w\geq 0 \tag{43}$$

To induce the lowest payment, this condition must be satisfied as an equality. Then, for the agent whose payment is the ith largest one, the following constraint must be satisfied:

$$(1-\beta_i)\left[v_i+(n-1)w-c\right]+\beta_i w-\frac{\alpha_i+\beta_i}{n-1}\sum_{k<i}\left[v_k-v_i\right]\geq 0 \tag{44}$$

To induce the lowest payment, this condition must be satisfied as an equality. Solving the resulting system of $(n-1)$ equalities as functions of $(v_2,...,v_n)$, we obtain the desired expressions.□

Proof of Proposition 8: We have:

$$v_1-v_2=\frac{\beta-(1-\beta)J_1}{1-\beta+\frac{\alpha_2+\beta}{N-1}}=0 \tag{45}$$

Then, using the expression of v_i for $i\geq 2$ it is easily checked that $v_i=v_{i-1}$ as $J_i=J_{i-1}=J_1$ is satisfied.□

Proof of Proposition 9: We obtain after straightforward computations:

$$v_{i-1}-v_i=\frac{(\beta_i-\beta_{i-1})+\frac{i-2}{n-1}(\beta_i-\beta_{i-1})\alpha+\frac{(\beta_i-\beta_{i-1})(1+\alpha)}{n-1}\left[\sum_{j=1}^{i-2}J_j\right]}{\left[1-\beta_i+\frac{i-1}{n-1}(\alpha+\beta_i)\right]\left[1-\beta_{i-1}+\frac{i-2}{n-1}(\alpha+\beta_{i-1})\right]} \tag{46}$$

We conclude that $v_{i-1}\geq v_i$ if and only if $\beta_i\geq\beta_{i-1}$ is satisfied.□

Proof of Proposition 10: Looking at the optimal ranking satisfying $v_{i-1}\geq v_i$ for any $i\in I$, we first obtain:

$$v_1 \geq v_2 \Longleftrightarrow \frac{\beta_2 + \frac{\alpha_2+\beta_2}{n-1} J_1}{1 - \beta_2 + \frac{\alpha_2+\beta_2}{n-1}} \geq J_1 \tag{47}$$

which, after simplification, is equivalent to:

$$\beta_2 \geq (1-\beta_2) J_1 = (1-\beta_2) \frac{\beta_1}{1-\beta_1} \tag{48}$$

or

$$\frac{\beta_2}{1-\beta_2} \geq \frac{\beta_1}{1-\beta_1} \tag{49}$$

which is equivalent to $\beta_2 \geq \beta_1$. We now move on to the case where $i \geq 3$ holds, and we have:

$$v_{i-1} \geq v_i \Longleftrightarrow \frac{\beta_i + \frac{\alpha_i+\beta_i}{n-1} \left[\sum_{l=1}^{i-1} J_l\right]}{1 - \beta_i + (i-1)\frac{\alpha_i+\beta_i}{n-1}} \geq J_{i-1} \tag{50}$$

Rewriting and simplifying the right hand side term of this equivalence, then using the expression of J_{i-1} as a function of $\sum_{l=1}^{i-2}$, we conclude that $v_{i-1} \geq v_i$ is equivalent to:

$$(\beta_i - \beta_{i-1}) + \frac{\sum_{l=1}^{i-2} J_l}{n-1} \left[(\alpha_i + \beta_i) - (\alpha_{i-1} + \beta_{i-1}) - \beta_{i-1}\alpha_i + \beta_i \alpha_{i-1}\right] + \frac{i-2}{n-1} \left[\beta_i \alpha_{i-1} - \beta_{i-1}\alpha_i\right] \geq 0. \tag{51}$$

Now, if agents' aversion to inequalities parameters are such that there exists a ranking satisfying $\frac{\beta_i}{\beta_{i-1}} > \frac{\alpha_i}{\alpha_{i-1}} \geq 1$ for any $i \geq 3$ then it satisfies $\alpha_i \geq \alpha_{i-1}$ and $\beta_i > \beta_{i-1}$ for any $i \geq 3$, and it satisfies condition (14).

Finally, assuming that this type of ranking does not exist, we provide an example where the optimal ranking necessarily satisfies $\alpha_i < \alpha_{i-1}$ for some $i \in I$. Indeed, let us consider $n = 3$: we know that the agent ranked first satisfies $\beta_1 \leq \beta_2$, and we here assume that this inequality is strict. Then we consider a situation where $\beta_2 = \beta_3 = \beta > \beta_1$. Using condition (14) for $i = 3$ yields that $v_2 \geq v_3$ is equivalent to:

$$\beta + (1-\beta)\frac{\alpha_3 + \beta}{n-1} J_1 + \frac{\beta}{n-1}\alpha_2 \geq \beta + (1-\beta)\frac{\alpha_2 + \beta}{n-1} J_1 + \frac{\beta}{n-1}\alpha_3 \tag{52}$$

Rewriting, we obtain:

$$\frac{(1-\beta)\beta_1}{(n-1)(1-\beta_1)} (\alpha_3 - \alpha_2) \geq \frac{\beta}{n-1} (\alpha_3 - \alpha_2) \tag{53}$$

If $\alpha_3 \geq \alpha_2$ this inequality is easily checked to be equivalent to $\beta_1 \geq \beta$, which contradicts our initial finding. As such, we conclude that $\alpha_3 < \alpha_2$ must necessarily hold.□

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